



# EUI WORKING PAPERS IN ECONOMICS

EUI Working Paper ECO No. 91/42

## Exchange Rates and Oligopoly

THORSTEN HENS, ALAN KIRMAN  
and  
LOUIS PHILIPS

European University Institute, Florence



European University Library



3 0001 0016 6687 6

*Please note*

As from January 1990 the EUI Working Paper Series is divided into six sub-series, each sub-series will be numbered individually (e.g. EUI Working Paper LAW No 90/1).

**EUROPEAN UNIVERSITY INSTITUTE, FLORENCE**

**ECONOMICS DEPARTMENT**

**EUI Working Paper ECO No. 91/42**

**Exchange Rates and Oligopoly**

**THORSTEN HENS, ALAN KIRMAN  
and  
LOUIS PHILIPS**



WP

330

EUR

**BADIA FIESOLANA, SAN DOMENICO (FI)**

All rights reserved.  
No part of this paper may be reproduced in any form  
without permission of the authors.

© Thorsten Hens, Alan Kirman and Louis Phlips  
Printed in Italy in June 1991  
European University Institute  
Badia Fiesolana  
I-50016 San Domenico (FI)  
Italy

## EXCHANGE RATES AND OLIGOPOLY

Thorsten Hens, University of Bonn,  
Alan Kirman and Louis Philips, European University Institute, Florence

May 1991.

### ABSTRACT

We consider two duopolistic firms which both operate in two countries. The markets of the two countries are separate and each of the firms produces its good in one of these countries. We study the effect of an exchange rate change on the prices in each country and on the level of sales and of profits of each of the firms. When strong restrictions such as constant marginal costs are imposed, prices move in the "right" direction in response to an exchange rate change. However, with general cost and demand structures, even in this simple model, it is possible for prices in both countries to move in "perverse" directions.



## Content

1. Introduction
2. The Model
3. Restrictions for Comparative Statics
4. Constant Marginal Costs (Independent Markets)
5. Diseconomies of Scope
  - 5.1. No Strategic Interaction (Independent Firms)
  - 5.2. Strategic Complements
  - 5.3. Strategic Substitutes
6. General Cost and Demand Functions
7. An Example
8. Conclusion
9. References

## Exchange Rates and Oligopoly

Thorsten Hens,

University of Bonn

Alan Kirman and Louis Phlips,

European University Institute, Florence

### 1. Introduction

One of the more important questions in open macroeconomics has been that of the magnitude and direction of the changes in prices resulting from a change in exchange rates. We will examine this question at the level of a market for one good and will emphasise the role of industry structure and market separation. We will show that unless very restrictive assumptions are made even the direction of price changes may be perverse. It used to be argued that a devaluation reduces the foreign currency price of exports and increases the domestic price of imports. The overall impact of devaluation on the balance of trade was held to depend on the structure of cost functions. This analysis clearly recognised the idea that domestic firms might not be perfect competitors on the foreign, or international market. This approach was superseded by the monetary approach, which argued that the consequence of a devaluation would automatically be offset by an inflationary process, which would restore prices in the domestic country to their former level in foreign currency terms (see Johnson (1972)). This argument justifies the Purchasing Power Parity theory of exchange rate determination and it is suggested that, in the long run, arbitrage would restore equilibrium. However, the precise mechanism through which this would be achieved is not spelled out.

The so-called "Scandinavian Analysis" (see Aukrust (1977)) tried to rectify this by considering the domestic economy as operating on a perfectly competitive

world market and suggesting that, after a devaluation, there would be pressure from workers to bid up wages to offset their loss in real income. The result would be that domestic prices would rise till the effect of the devaluation was just offset. Thus the basic argument is that domestic prices adjust so as to remove the differential induced by a change in the exchange rate, and not that the differential is consciously chosen. Indeed, at the end of the 1970s, Robinson et al. (1979) claimed that: "Under fixed exchange rates relative prices were stable, and changes in the exchange rate generated offsetting movements in the domestic currency prices of exports, so that the price in foreign currency terms did not change."

Experience in the 1980s has done a lot to undermine this view. As Fisher (1989) points out, in that period the variance of the monthly changes of the Federal Reserve Board's dollar index was more than 6%, whilst the dollar price of imports has been much less volatile. Even if this could be reconciled, in the short run, with the idea that markets are competitive, it would be difficult to explain why the appreciation of the dollar in the early '80s did not lead to a proportionate decrease in import prices in the U.S. (see Mann (1986) and Feinberg (1986)). Indeed, the very fact that exchange rate changes are not fully passed through may, in part, explain the volatility of these exchange rates which are failing to achieve the adjustments discussed previously.

These observations have led international trade theory to examine the role of market structure, at least in the home market (the U.S.). In the seminal paper by Dornbusch (1987), oligopoly and/or monopolistic competition in the home market is shown to imply an incomplete pass-through of a change in the exchange rate. Price discrimination by foreign firms is taken into consideration by Krugman (1987), Giovannini (1988), Feenstra (1989) and Knetter (1989). The response of primary commodity prices is discussed in Gilbert (1991).

The Dornbusch results are obtained using particular specifications of the demand and cost functions for firms of equal size. The assumption of constant marginal production costs allows one to study the impact of exchange rate changes



on the home market price, ignoring prices and sales in the foreign market. This paper tries to generalise the Dornbusch results in several directions. First, particular choices of the form of demand and cost functions are avoided. (It is well known, and indeed shown, that oligopolistic equilibria are sensitive to such specifications.) Second, the home and the foreign market equilibria are analysed simultaneously, with oligopoly in both markets. Third, price discrimination is allowed for, since oligopolists selling in two markets with different demands will find it profitable to price-discriminate as long as markets are separated. (We suppose that there are no tariff barriers nor transportation costs, but that market separation results from marketing devices (such as exclusive dealing) or legal and administrative barriers. Our approach is thus well suited for the analysis of trade between two member countries of the EEC.) Fourth, the production technology is nonlinear: economies and diseconomies of scale, as well as joint economies and diseconomies of scope (over the two markets) may be present. Finally, firm size can differ.

We adopt the comparative statics for oligopoly methodology developed by Dixit (1986). The analysis is therefore static, and inevitably suffers from the limitations inherent in comparative statics.

Section 2 presents the model. Section 3 analyses what a priori restrictions for comparative statics are imposed by the model. In section 4 we show that, with constant marginal costs, an appreciation of the foreign currency decreases the foreign price and increases the domestic price, and that the exchange rate changes are not fully passed through. We then introduce diseconomies of scope and show in section 5 that, with strategic complements, prices react as in section 4. However, except for the special case of identical firms, when there are strategic substitutes it is possible that both prices move in the same direction. As section 6 shows, with general cost functions we may even get the perverse result that an appreciation of the foreign currency leads to an increase in the foreign price and decreases the domestic price. Section 7 illustrates our results by an example.

## 2. The Model

We consider two markets, market 1 and market 2, separated by barriers other than tariffs and transportation costs. We examine the case in which there is a duopolistic structure, i.e. there is one firm located in market 1, selling  $x_{11}$  in market 1 and  $x_{12}$  in market 2 and there is one firm located in market 2, selling  $x_{21}$  in market 1 and  $x_{22}$  in market 2. The commodity is homogeneous. Generalising to the case in which there are  $m$  firms in the first market and  $n$  firms in the second complicates the notation considerably, and does not change the results in the first part of the paper. In the second part, it will be seen that it is sufficient for the two firms to be different for us to be able to obtain perverse results. Thus adding more firms would simply reinforce this and the outcomes can only be restricted by making all firms symmetric. Limiting ourselves to the duopolistic case does not therefore restrict our results and greatly simplifies the presentation.

The profit function of the firm located in market 1, expressed in market 1 currency, is

$$\Pi_1 = P_1(X_1)x_{11} + eP_2(X_2)x_{12} - c_1(x_{11}, x_{12}) \quad (1)$$

where

$$X_1 = \sum_{i=1}^2 x_{i1} \quad \text{and} \quad X_2 = \sum_{i=1}^2 x_{i2}.$$

The inverse market demand functions are  $P_1(X_1)$  and  $P_2(X_2)$ , respectively. The exchange rate  $e$ , is the value in market 1 currency of the currency used in market 2. (Thus were the "law of one price" to hold it would say that the prices of the commodity must be such that  $P_1 = eP_2$ ). The cost functions are  $c_i(x_{i1}, x_{i2})$ , with marginal costs  $c_i^1$  and  $c_i^2$ . Superscripts denote derivatives with respect to the first, respectively second, argument.

The firm located in market 2 has the profit function

$$\Pi_2 = P_1(X_1)x_{21} + eP_2(X_2)x_{22} - ec_2(x_{21}, x_{22}) \quad (2)$$

defined in market 1 currency. Thus our market is described by  $\mathcal{E} = (P_1, P_2, c_1, c_2)$  and  $e$ .

We shall make the following assumptions concerning the demand functions, the cost functions and the resultant profit functions.

**A1** The inverse demand functions  $P_j(X_j)$ ,  $j = 1, 2$  are continuous for all  $X_j > 0$ . For each  $j$  there exists  $\bar{X}_j > 0$  such that  $P_j(X_j) = 0$  for all  $X_j \geq \bar{X}_j$  and  $P_j(X_j) > 0$  for  $X_j < \bar{X}_j$ . Furthermore,  $P_j(0) = \bar{P}_j < \infty$  and for all  $X_j$  such that  $0 < X_j < \bar{X}_j$ ,  $P_j(X_j)$  has a continuous second derivative  $P_j''$  and  $P_j'(X_j) < 0$  for all  $X_j$ .

**A2** The cost function of the  $i$ th firm  $c_i(x_{i1}, x_{i2})$  is defined and continuous for all output levels  $x_{i1} \geq 0$ ,  $x_{i2} \geq 0$ .  $c_i(0, 0) \geq 0$ , and  $c_i$  has continuous first and second partial derivatives for all  $x_{i1}, x_{i2} \geq 0$ . Furthermore,  $c_i^1 > 0$  and  $c_i^2 > 0$  for all  $x_{i1} \geq 0$  and  $x_{i2} \geq 0$ .

**A3** For all  $x_{i1}, x_{i2} > 0$ ,  $X_1 < \bar{X}_1$  and  $X_2 < \bar{X}_2$ .  $\Pi_i(x_{i1}, x_{i2}, X_1, X_2)$  is concave.

We will be considering a *Cournot Nash solution* in which each firm takes the other firms' strategies as given. With this in mind, let us look at the maximising behaviour of the firms.

Differentiating  $\Pi_1$  and  $\Pi_2$ , the marginal profits are

$$\Pi_1^1(x_{11}, x_{12}, X_1, X_2) = P_1'(X_1)x_{11} + P_1(X_1) - c_1^1(x_{11}, x_{12}) \quad (3)$$

$$\Pi_1^2(x_{11}, x_{12}, X_1, X_2) = e(P_2'(X_2)x_{12} + P_2(X_2)) - c_1^2(x_{11}, x_{12}) \quad (4)$$

for firm 1, and

$$\Pi_2^1(x_{21}, x_{22}, X_1, X_2) = P_1'(X_1)x_{21} + P_1(X_1) - e c_2^1(x_{21}, x_{22}) \quad (5)$$

$$\Pi_2^2(x_{21}, x_{22}, X_1, X_2) = e(P_2'(X_2)x_{22} + P_2(X_2) - c_2^2(x_{21}, x_{22})) \quad (6)$$

for firm 2.



Thus the system of equations

$$\Pi_i^j(x_{i1}^*, x_{i2}^*, X_1^*, X_2^*) = 0 \quad i, j = 1, 2 \quad (7)$$

describes the first-order equilibrium conditions.

The second-order conditions are

$$A_{ii} := \begin{bmatrix} \Pi_i^{11} & \Pi_i^{12} \\ \Pi_i^{21} & \Pi_i^{22} \end{bmatrix} \text{ negative definite, } i = 1, 2$$

which are assured by the concavity of  $\Pi_i$ ,  $i = 1, 2$  (assumption A3).

### Equilibrium

We are, as we have said, interested here in a *Cournot Nash equilibrium*, i.e. a situation in which once the strategies of all players are specified, no individual has an incentive to modify his own. We have, in effect, already defined the *reaction functions* of each firm  $i$ , i.e.

$$\Gamma_i = \operatorname{argmax} \Pi_i(x_{i1}, x_{i2}, X_1, X_2) \quad i = 1, 2$$

and an *equilibrium* is therefore a *fixed point* of the mapping

$$\Gamma : \mathbb{R}_+^4 \rightarrow \mathbb{R}_+^4.$$

It is easily proved that, given A1, A2 and A3, equilibrium exists (see Friedman (1977)). Indeed, if  $P_j' < 0$ ,  $P_j'' < 0$  and  $c_i'' > 0$  for both countries and both firms, then A3 is satisfied and existence is ensured. However, if the second term is positive, i.e. if demand is convex and the cost function is concave (increasing returns to scale) A3 may still hold if the first term is large enough. Since we are interested in considering the case of increasing returns, this is important in what follows.

Our model does not yet have enough structure for us to be able to establish the consequences of an exchange rate change. We will thus make the assumption that the model is stable with respect to a natural adjustment process. This process

embodies the idea that a firm will increase its output if it obtains a positive marginal profit from so doing.

Thus we may write the profit adjustment process as

$$\dot{x} = \mu(x, e) \quad (7)$$

where

$$x = \begin{pmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} \Pi_1^1 \\ \Pi_1^2 \\ \Pi_2^1 \\ \Pi_2^2 \end{pmatrix}$$

Linearising around the equilibrium point  $x^*$ , i.e. taking a first order Taylor expansion, we obtain

$$\begin{pmatrix} \dot{x}_{11} \\ \dot{x}_{12} \\ \dot{x}_{21} \\ \dot{x}_{22} \end{pmatrix} = \begin{bmatrix} a_{11} & -c_1^{12} & b_{11} & 0 \\ -c_1^{21} & a_{22} & 0 & eb_{12} \\ b_{21} & 0 & a_{33} & -ec_2^{12} \\ 0 & eb_{22} & -ec_2^{21} & a_{44} \end{bmatrix} \cdot \begin{pmatrix} x_{11} - x_{11}^* \\ x_{12} - x_{12}^* \\ x_{21} - x_{21}^* \\ x_{22} - x_{22}^* \end{pmatrix} \quad (8)$$

where

$$b_{ij} := P_j''(X_j^*)x_{ij}^* + P_j'(X_j^*) \quad i, j = 1, 2 \quad (9)$$

which represents the change in the marginal profit of firm  $i$  in market  $j$  (in currency  $j$ ) when the other firm increases its sales in that market, and

$$a_{11} = b_{11} + P_1'(X_1^*) - c_1^{11}(x_{11}^*, x_{12}^*) \quad (10)$$

$$a_{22} = e(b_{12} + P_2'(X_2^*)) - c_1^{22}(x_{11}^*, x_{12}^*) \quad (11)$$

$$a_{33} = b_{21} + P_1'(X_1^*) - ec_2^{11}(x_{21}^*, x_{22}^*) \quad (12)$$

$$a_{44} = e(b_{22} + P_2'(X_2^*) - c_2^{22}(x_{21}^*, x_{22}^*)). \quad (13)$$

In short we write (8) as  $\dot{x} = A_{(x^*)}(x - x^*)$ . The coefficient matrix  $A$  clearly satisfies a necessary condition for stability, i.e. that the trace be negative, since this follows from A3. But to justify using comparative statics we have to go a step further and assume that  $\dot{x} = \mu(x^*, e)$  is locally asymptotically stable.

**A4.1** All eigenvalues of  $A_{(x^*)}$  have negative real parts.

Since A4.1 is not always a convenient assumption to work with, we will replace it sometimes with

**A4.2**  $A$  is a negative Hadamard matrix, i.e.

$$a_{ii} < 0 \text{ and } |a_{ii}| > \sum_{j \neq i} |a_{ij}|, \quad i = 1, \dots, 4$$

which is a much stronger condition than stability.

### 3. Restrictions for Comparative Statics

We are interested in how the equilibrium given by the system of equations (7) reacts to exchange rate changes. At a first glance, one might guess that an appreciation of currency 2 decreases  $P_2$  and increases  $P_1$ . We will call this the “normal” reaction. If both prices move in the same direction, we say the exchange rate change leads to a “surprising” reaction, and if  $P_2$  increases and  $P_1$  decreases we will call this a “perverse” reaction. To see what is going on in our model, we first analyse the way in which the various forces (effects) act. The interactions are illustrated in figure 1.

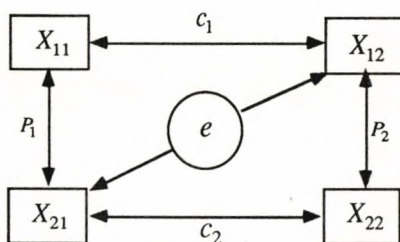


Figure 1

An increase in  $e$  has two direct effects. That is to say, *ceteris paribus*, selling in market 2 becomes more attractive to firm 1, i.e.  $x_{12}$  increases and selling in market 1 becomes less attractive for firm 2, i.e.  $x_{21}$  decreases. Then these



direct effects induce cost and demand effects. Depending on whether  $c_1$  exhibits economies of scope ( $c_1^{12} < 0$ ) or diseconomies of scope ( $c_1^{12} > 0$ ) the increase in  $x_{12}$  will increase (resp. decrease)  $x_{11}$ . Analogous arguments apply to  $c_2$ . An increase in  $x_{12}$  decreases (increases)  $x_{22}$  if  $x_{22}$  is a strategic substitute ( $b_{22} < 0$ ) resp. a strategic complement ( $b_{22} > 0$ ) of  $x_{12}$ . And these secondary effects induce further effects and so on.<sup>1</sup> To see the overall impact on the system we have to solve the system of equations

$$A_{(x^*)}dx = \gamma_{(x^*)}de \quad (14)$$

for  $dx$ , where

$$\gamma_{(x^*)} = \begin{pmatrix} 0 \\ \gamma_1 \\ \gamma_2 \\ 0 \end{pmatrix} \quad \text{with} \quad \begin{aligned} \gamma_1 &= (-1/e)c_1^2(x_{11}^*, x_{12}^*), \\ \gamma_2 &= c_2^1(x_{21}^*, x_{22}^*). \end{aligned}$$

Without risk of confusion we will refer from now on to the elements of  $A_{(x^*)}$  as  $a_{ij}$ . The next lemma clarifies which restrictions on the comparative statics equations (14) are implied by the assumptions made so far. Before we state the lemma, we remark that our assumptions A1 - A3 lead to the following restrictions for (14). (A4 is a direct assumption on  $A_{(x^*)}$  thus, we do not mention it in this context).

From the definition of the profit functions, we must have that

$$(i) \quad a_{14} = a_{23} = a_{32} = a_{41} = 0 \text{ and } a_{12} = a_{21}, a_{34} = a_{43}.$$

Furthermore A2 restricts

$$(ii) \quad \gamma_1 \text{ to be negative and } \gamma_2 \text{ to be positive.}$$

Finally, A3 means that

$$(iii) \quad A_{11} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{and} \quad A_{22} = \begin{bmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{bmatrix} \text{ are negative definite.}$$

But apart from these three conditions,  $A_{(x^*)}$  may take any values, provided that the following regularity condition is satisfied.

<sup>1</sup> A full discussion of this type of problem is given by Bulow, Geanakoplos and Klemperer (1985), who introduced the notions of "strategic" complements and substitutes.

$$(iv) \ a_{13} \neq a_{31}, a_{24} \neq a_{42}$$

**Lemma 1.** *Let  $e > 0$  be any exchange rate and let  $(A, \gamma)$  be any set of parameters for a  $4 \times 4$  linear system of equations that satisfies (i) - (iv) given above. Then there exists an International Market  $\mathcal{E} = (c_i, P_j) \ i, j = 1, 2$  satisfying assumptions A1 - A3 such that  $(A, \gamma)$  is generated by  $\mathcal{E}$  as the parameter values of its comparative statics equation (14).*

PROOF:

Since only the equilibrium values of  $A(\ )$  and  $\gamma(\ )$  matter in (14), at the point  $x^*$ , we can consider the values of the functions  $(c_i, P_j) \ i, j = 1, 2$  and their first and second derivatives as the parameters which we are free to choose. Together with  $x^*$  itself, we then have 22 parameters to generate the 12 different values given by  $(A, \gamma)$ .

To satisfy A1 and A2 we only have to restrict our choice to values  $(x_{ij}^* > 0, c_i(x_{i1}^*, x_{i2}^*) > 0, c_i'(x_{i1}^*, x_{i2}^*) > 0, P_j(X_j^*) > 0$  and  $P_j'(X_j^*) < 0) \ i, j = 1, 2$  and extend  $(c_i, P_j) \ i, j = 1, 2$  to points different from  $x^*$  appropriately. Since  $A_{11}$  and  $A_{22}$  are negative definite, we can always extend  $(c_i, P_j) \ i, j = 1, 2$  in such a way that A3 is satisfied.

To simplify notation, in the following, we use the function symbols to denote the values of the functions at  $x^*$ , e.g.  $P_j$  denotes  $P_j(X_j^*) \ j = 1, 2$ .

To sum up then, the problem is to find  $(x_{ij}^* > 0, c_i > 0, c_i' > 0, P_j > 0, P_j' < 0$  and  $c_i^{rs}, r, s = 1, 2, P_j'' \ i, j = 1, 2$  such that

$$(15.1) \ P_1 x_{11}^* + P_2 x_{12}^* - c_1 > 0 \quad i = 1, 2$$

$$(15.2) \ P_1' x_{11}^* + P_1 - c_1^1 = 0$$

$$(15.3) \ e(P_2' x_{12}^* + P_2) - c_1^2 = 0$$

$$(15.4) \ P_1' x_{21}^* + P_1 - e c_2^1 = 0$$

$$(15.5) \ P_2' x_{22}^* + P_2 - c_2^2 = 0$$

$$(15.6) \ P_1'' x_{11}^* + P_1' = b_{11}$$

$$(15.7) \ P_2'' x_{12}^* + P_2' = b_{12}$$

$$(15.8) \ P_1'' x_{21}^* + P_1' = b_{21}$$

$$(15.9) \ P_2'' x_{22}^* + P_2' = b_{22}$$

$$(15.10) \ b_{11} + P_1' - c_1^{11} = a_{11} < 0$$

$$(15.11) \ e(b_{12} + P_2') - c_1^{22} = a_{22} < 0$$

$$(15.12) \ b_{21} + P_1' - e c_2^{11} = a_{33} < 0$$

$$(15.13) \quad e(b_{22} + P_2' - c_2^{22}) = a_{44} < 0$$

$$(15.14) \quad c_1^2 = -e\gamma_1$$

$$(15.15) \quad c_2^1 = \gamma_2$$

$$(15.16) \quad -c_1^{12} = a_{12} = -c_1^{21}$$

$$(15.17) \quad -ec_2^{12} = a_{34} = -ec_2^{21}$$

Now, choose  $c_2^{21} = c_2^{12}$ ,  $c_1^{12} = c_1^{21}$  and  $c_2^1$ ,  $c_1^2$  to satisfy (15.14) - (15.17).

We see that given any  $(b_{ij}, P_j')$   $i, j = 1, 2$  we can find  $(c_i^{jj})$   $i, j = 1, 2$  to satisfy (15.10) - (15.13). Furthermore there always exist  $c_i > 0$  such that (1) is still satisfied since  $(P_j > 0, x_{ij}^* > 0)$   $i, j = 1, 2$ .

Thus we are left with (15.2) - (15.9) and the parameters

$$(x_{ij}^* > 0, c_i^i > 0, P_j > 0, P_j' < 0, P_j'') \quad i, j = 1, 2$$

Given any  $(P_j' < 0$  and  $x_{ij}^* > 0$   $i \neq j$ )  $i, j = 1, 2$  we find  $(c_i^i > 0, x_{ii}^* > 0$  and  $P_i > 0)$   $i = 1, 2$  to satisfy (15.2) - (15.5):

Since  $(P_j' < 0$  and  $(x_{ij}^* > 0$  and  $c_i^i > 0)$   $i \neq j$ )  $i, j = 1, 2$  we can always find  $P_j > 0$ ,  $j = 1, 2$  to satisfy (15.3) and (15.4).

To fulfil (15.2) first choose  $x_{11}^* > 0$  small enough to make  $P_1'x_{11}^* + P_1$  positive and then choose  $c_1^1 > 0$  appropriately. Proceed analogously for (15.4).

Thus we are left with  $(P_j' < 0, P_j'', x_{ij}^*, i \neq j)$   $i, j = 1, 2$  for (15.6) - (15.9).

Solving (15.6) and (15.8) as a linear system in  $P_1'$  and  $P_1''$  we get  $P_1' = \frac{b_{21}x_{11}^* - b_{11}x_{21}^*}{x_{11}^* - x_{21}^*}$ .

Thus, since  $b_{21} \neq b_{11}$  it is always possible to choose  $x_{21}^*$  such that  $P_1' > 0$  solves (15.6) and (15.8). Now do the same for (15.7) and (15.9) and this completes the proof. ■



#### 4. Constant Marginal Costs (Independent Markets)

In this section we make the assumption that the marginal costs of both firms are constant. This considerably simplifies the analysis since the feedback from one market to another through costs is eliminated. Each firm can effectively solve its problem on each market separately. A change in the quantity sold on one market will not affect the marginal cost of the quantity sold on the other. With this assumption we get a definite answer to how prices react to exchange rate changes:

**Proposition 1.** *Under A1 - A4.1 in the case of constant marginal costs, an appreciation of the currency of market 2 decreases  $P_2$  and increases  $P_1$ . Furthermore, firm 1 increases sales in market 2 and firm 2 decreases sales in market 1, but firms' quantity changes in their home market depend on the strategic effect on marginal profits. If in the home market a firm's product is a strategic substitute (complement) to its opponent's products then its quantity changes in the opposite (same) direction as its opponent's quantity. The absolute value of the price elasticities is smaller than 1/3 (incomplete pass through) if demand functions are linear.*

PROOF:

If in the system of equations (14) we exchange the second equation with the third equation, we see that with constant marginal costs markets are independent, i.e. (14) decompose into two independent 2x2 linear systems:

$$\underbrace{\begin{bmatrix} b_{11} + P'_1 & b_{11} & 0 & 0 \\ b_{21} & b_{21} + P'_1 & 0 & 0 \\ 0 & 0 & e(b_{12} + P'_2) & eb_{12} \\ 0 & 0 & eb_{22} & e(b_{22} + P'_2) \end{bmatrix}}_{\tilde{A}} \cdot \begin{pmatrix} dx_{11} \\ dx_{21} \\ dx_{12} \\ dx_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ \gamma_2 \\ \gamma_1 \\ 0 \end{pmatrix} de \quad (16)$$

The solution is:

$$\frac{dx_{11}}{de} = \frac{-\gamma_2 b_{11}}{|A_{11}|}, \quad \frac{dx_{12}}{de} = \frac{\gamma_1 (b_{22} + P'_2) e}{|A_{22}|}$$

$$\frac{dx_{21}}{de} = \frac{\gamma_2 (b_{11} + P'_1)}{|A_{11}|}, \quad \frac{dx_{22}}{de} = \frac{-\gamma_1 b_{22} e}{|A_{22}|}.$$

The signs of the terms involved are

$$\gamma_1 < 0, \quad \gamma_2 > 0 \quad \text{by A2}$$

$$(b_{ii} + P'_i) < 0, \quad i = 1, 2 \quad \text{by A3}$$

$$|\tilde{A}_{ii}| > 0, \quad i = 1, 2 \quad \text{by A4}$$

thus the second part of proposition 1 is proved.

For market totals we get

$$\frac{dX_1}{de} = \frac{\gamma_2 P'_1}{|\tilde{A}_{11}|} < 0 \quad \text{and} \quad \frac{dX_2}{de} = \frac{\gamma_1 P'_2 \epsilon}{|\tilde{A}_{22}|} > 0 \quad \text{by A1}$$

Thus, since  $\frac{dP_j}{de} = P'_j \frac{dX_j}{de}$ , this proves the first part.

The elasticities are given by  $\epsilon_{P_j \epsilon} := \frac{dP_j}{de} \frac{\epsilon}{P_j} = P'_j \frac{dX_j}{de} \frac{\epsilon}{P_j} \quad j = 1, 2$ . Computation of these values for linear demand functions using the results derived so far, and taking into account the first order conditions (7), gives

$$\epsilon_{P_1 \epsilon} = \frac{p'_1 x_{21} + P_1}{3 P_1} \quad \text{and} \quad \epsilon_{P_2 \epsilon} = \frac{p'_2 x_{12} + P_2}{3 P_2}$$

Since by (7) and A2 the numerator of these expressions must be positive, from A1 we finally conclude

$$-1/3 < \epsilon_{P_2 \epsilon} < 0 < \epsilon_{P_1 \epsilon} < 1/3$$

which completes the proof. ■

Thus in this particular case we see that prices move in the standard direction. However it is interesting to note that the extent to which exchange rates are passed through is very limited if demand curves are linear. This gives an indication of the non-linearity in demand that would be required to move the elasticity of prices with respect to exchange rate changes towards one. Note also that from the proof it is easy to see that if the equilibrium were unstable all quantities would move in the opposite direction and the results would be reversed.

## 5. Diseconomies of Scope

### 5.1. No Strategic Interaction (Independent Firms)

We first investigate the case in which strategic interaction can be neglected, that is where the  $b_{ij}$  are arbitrarily small. Of course, every market with diseconomies of scope can only generate parameters  $A, \gamma$  which in addition to (i) - (iv) satisfy (v)  $a_{12} < 0$  and  $a_{34} < 0$ . But this is the only additional restriction imposed by diseconomies of scope. Thus without additional knowledge of the market  $\mathcal{E} = (c_i, P_j) \quad i, j = 1, 2$  the case of no strategic interaction is indeed justified. One might think it is natural that diseconomies of scope come together with diseconomies of scale and that for these markets no strategic interaction is hard to justify. But even for this case we can still prove that (i) - (v) are the only restrictions for  $A, \gamma$ . One only has to recognize that  $|P_j'| \quad j = 1, 2$  solving (15.6) - (15.9) can be chosen to be arbitrarily small, so that  $|b_{ij}| \quad i, j = 1, 2$  small does not conflict with  $c_i^{i,j} > 0 \quad i, j = 1, 2$  in (15.10) - (15.13).

In proposition 2 we do not need to assume stability of the adjustment process since in the case of independent firms, this is implied by the concavity of the firms' profit functions. More importantly, however, the removal of the strategic interaction consideration does, as one might expect, allow the changes in individual firms' quantities to become predictable and this is seen from

**Proposition 2.** *Given A1 - A3 with diseconomies of scope both firms increase sales in the market whose currency appreciates and decrease sales in the other market if strategic interaction can be neglected.*

PROOF: By A3,  $|A(x^*)| \neq 0$  for  $|b_{ij}| \quad i, j = 1, 2$  small.

Since the solution to (14) is a continuous function of the parameters  $A_{(x^*)}, \gamma(x^*)$  as long as  $|A_{(x^*)}| \neq 0$ , instead of taking the limit of the general solution for  $|b_{ij}| \rightarrow 0 \quad i, j = 1, 2$  we can as well solve (14) for the limit case  $b_{ij} = 0 \quad i, j = 1, 2$ . Then the problem decomposes into the  $2 \times 2$  linear systems.

$$A_{11}(x^*) \begin{pmatrix} dx_{11} \\ dx_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ \gamma_1(x^*) \end{pmatrix} de \quad \text{and} \quad A_{22}(x^*) \begin{pmatrix} dx_{12} \\ dx_{22} \end{pmatrix} = \begin{pmatrix} \gamma_2(x^*) \\ 0 \end{pmatrix} de$$



The solution is:

$$\frac{dx_{11}}{de} = \frac{-\gamma_1 a_{12}}{|A_{11}|} < 0, \frac{dx_{12}}{de} = \frac{\gamma_1 a_{11}}{|A_{11}|} > 0, \frac{dx_{21}}{de} = \frac{\gamma_2 a_{44} e}{|A_{22}|} < 0, \frac{dx_{22}}{de} = \frac{-\gamma_2 a_{43} e}{|A_{22}|} > 0$$

which completes the proof. ■

## 5.2. Strategic Complements

If goods are strategic complements and production exhibits diseconomies of scope then the iterative adjustment process  $x_{t+1} = \max\{0, x_t + \mu(x_t, e)\}$   $t = 1, 2, 3, \dots$  never has counter effects. That is to say, if  $e$  increases, in the first step  $x_{12}$  increases and  $x_{21}$  decreases. In the second step the cost and strategic effects lead to a decrease in  $x_{11}$  and an increase in  $x_{22}$ . All the following steps reinforce these changes. Nevertheless the process is stable if the slopes of the demand curves,  $|P'_j|$   $j = 1, 2$ , are big enough. Figure 2, which has to be read with reference to figure 1, shows these effects.

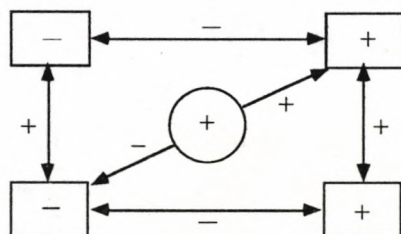


Figure 2

In figure 2, a + sign (- sign) in a box or a circle means that the variable given in figure 1 increases (decreases). A + sign (- sign) at an arrow from one variable to another means that an increase of the first leads to an increase (a decrease) of the second.

**Proposition 3.** *Given  $A1 - A4.2$  with diseconomies of scope both firms increase sales in the market whose currency appreciates and decrease sales in the other market if goods are strategic complements.*

PROOF:

First market:

Solving (14) for  $dx_{11}$  and  $dx_{21}$  leads to

$$|A| \frac{dx_{11}}{de} = -\gamma_1 (a_{12}|A_{22}| + a_{13}a_{34}a_{42}) + \gamma_2 (a_{13}a_{24}a_{42} - a_{12}a_{24}a_{43} - a_{13}a_{22}a_{44})$$

$$|A| \frac{dx_{21}}{de} = -\gamma_1 (-a_{11}a_{34}a_{42} - a_{12}a_{31}a_{44}) + \gamma_2 (a_{11}a_{22}a_{44} - a_{11}a_{24}a_{42} - a_{12}a_{21}a_{44})$$

The signs of the expressions involved are

$$-\gamma_1 > 0 \quad \text{and} \quad \gamma_2 > 0 \quad \text{by A2}$$

$$a_{ii} < 0 \quad i = 1, 4 \quad \text{by A3 or A4.2}$$

$$a_{12} = a_{21} < 0 \quad \text{and} \quad a_{34} = a_{43} < 0 \quad \text{by diseconomies of scope}$$

$$a_{13} > 0, a_{24} > 0, a_{31} > 0 \quad \text{and} \quad a_{42} > 0 \quad \text{by strategic complements}$$

$$|A_{ii}| > 0 \quad i = 1, 2 \quad \text{by A3}$$

$$|A| > 0 \quad \text{by A4.2}$$

Thus we have only to consider the factors by which  $\gamma_2$  is multiplied.

We have

$$\begin{aligned} & a_{13}a_{24}a_{42} - a_{12}a_{24}a_{43} - a_{13}a_{22}a_{44} \\ &= a_{13}(a_{24}a_{42} - a_{22}a_{44}) - a_{12}a_{24}a_{43} < 0 \end{aligned} \quad \text{by A4.2}$$

and

$$\begin{aligned} & a_{11}a_{22}a_{44} - a_{11}a_{24}a_{42} - a_{12}a_{21}a_{44} \\ &= (a_{11}a_{22} - a_{12}a_{21})a_{44} - a_{11}a_{24}a_{42} \\ &< -(a_{11}(a_{22} + a_{24}) - a_{12}a_{21})a_{42} \quad \text{by A4.2} \\ &< -(a_{11} - a_{12})a_{21}a_{42} < 0 \quad \text{by A4.2} \end{aligned}$$

Second market:

For  $-|A| \frac{dx_{i2}}{de} \quad i = 1, 2$  we get analogous expressions. We only have to exchange  $-\gamma_1$  and  $\gamma_2$ ,  $a_{11}$  and  $a_{44}$ ,  $a_{22}$  and  $a_{33}$ ,  $a_{13}$  and  $a_{42}$ ,  $a_{12}$  and  $a_{43}$ ,  $a_{21}$  and  $a_{34}$ ,  $a_{24}$  and  $a_{31}$ ,  $a_{42}$  and  $a_{13}$ .

■

### 5.3. Strategic Substitutes

Up to this point we have found in all the cases that we have considered that an exchange rate change leads to a "normal" reaction; the price in the market whose currency appreciates decreases and the other price increases. The reason for this is that so far we have analysed cases in which only one of the indirect effects occurred or in which they both moved in the same direction. With diseconomies of scope and strategic substitutes it may happen that both prices move in the same direction but it is impossible that the price in the market whose currency appreciates increases and that the other prices decreases.

**Proposition 4.** *Given A1 - A3 with diseconomies of scope it cannot happen that a price increase in the market whose currency appreciates occurs together with a price decrease in the market of the depreciated currency, if goods are strategic substitutes.*

PROOF:

Assume the contrary, i.e.  $\frac{dX_1}{de} > 0$  and  $\frac{dX_2}{de} < 0$ . Now look at the first order profit maximizing condition for firm 1 and 2, or equivalently split up (14) into the first two and the last two equations. Since in the profit function of firm  $i$  the output levels of firm  $j$  ( $j \neq i$ ) matter only through the aggregate output levels we must have that

$$A_{11} \begin{pmatrix} dx_{11} \\ dx_{12} \end{pmatrix} = \begin{pmatrix} -b_{11}dX_1 \\ \gamma_1 de - eb_{12}dX_2 \end{pmatrix} \quad \text{and} \quad A_{22} \begin{pmatrix} dx_{21} \\ dx_{22} \end{pmatrix} = \begin{pmatrix} \gamma_2 de - b_{21}dX_1 \\ b_{22}dX_2 \end{pmatrix}$$

Given A2, A3, diseconomies of scope, strategic substitutes and  $de > 0$ ,  $dX_1 > 0$ ,  $dX_2 > 0$  the "sign structures" of these equations are

$$\begin{bmatrix} - & - \\ - & - \end{bmatrix} \begin{pmatrix} dx_{11} \\ dx_{12} \end{pmatrix} = \begin{pmatrix} + \\ - \end{pmatrix} \quad \text{rsp.} \quad \begin{bmatrix} - & - \\ - & - \end{bmatrix} \begin{pmatrix} dx_{21} \\ dx_{22} \end{pmatrix} = \begin{pmatrix} + \\ - \end{pmatrix}$$

Thus only  $|A_{ii}|dx_{i1} < 0$  and  $|A_{ii}|dx_{i2} > 0$   $i = 1, 2$  are compatible with these assumptions. But because of A3 we have  $|A_{ii}| > 0$   $i = 1, 2$  which contradicts  $\frac{dX_1}{de} > 0$  and  $\frac{dX_2}{de} < 0$ .

■



The next proposition shows that the existence of diseconomies of scope and strategic substitutes does not, however, rule out both prices moving in the same direction. We get this “surprising” result for markets in which the direct effect  $\gamma_2$  is relatively small compared to the direct effect  $\gamma_1$  and in which firm 1’s indirect cost effects are negligible. This of course is possible only if firms are quite different. But as our discussion before proposition 2 showed, even with diseconomies of scope, we cannot rule out this sort of asymmetry. Proposition 6 will show that different firms are necessary to get this “surprising” result. Figure 3 shows which effects we have to expect.

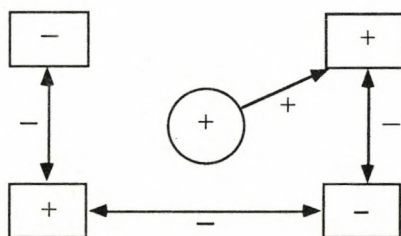


Figure 3

Figure 3 suggests that  $x_{12}$  and  $x_{21}$  increase whereas  $x_{11}$  and  $x_{22}$  decrease. But since the effects involved have a hierarchical ( tree ) structure one might expect that the increase in  $x_{12}$  would outweigh the decrease in  $x_{22}$  and that the increase of  $x_{21}$  would outweigh the decrease in  $x_{11}$ .

**Proposition 5.** *Given A1 - A4.2 with diseconomies of scope and a relatively small direct effect for firm  $i$ , i.e.  $|\frac{\gamma_i}{\gamma_j}|_{j \neq i}$  small and a negligible cost effect for firm  $j$  ( $j \neq i$ ), an appreciation of currency  $i$  increases both prices if goods are strategic substitutes.*

PROOF:

Without loss of generality take  $i = 2, j = 1$ .

A4.2 guarantees that  $|A_{(x^*)}| \neq 0$ . Since the solution to (14) is a continuous function of the parameters  $A_{(x^*)}, \gamma(x^*)$  instead of taking the limit of the general solution for

$|\frac{\gamma_2}{\gamma_1}| \rightarrow 0$  and  $|c_1^{21}| \rightarrow 0$  we can simply solve (14) for the limit case  $\gamma_2 = 0, c_1^{21} = 0$ . Because of proposition 4 it remains to show that  $\frac{dX_1}{de} > 0$  or  $\frac{dX_2}{de} > 0$ . Solving (14) for the first market we get

$$|A| \frac{dx_{11}}{de} = -\gamma_1 a_{13} a_{34} a_{42} \quad , \quad |A| \frac{dx_{21}}{de} = \gamma_1 a_{11} a_{34} a_{42}$$

Thus

$$|A| \frac{dX_1}{de} = \gamma_1 (a_{11} - a_{13}) a_{34} a_{42} > 0$$

by A2, A4.2, diseconomies of scope and strategic substitutes. ■

## 6. General Cost and Demand Functions

Until now we have imposed additional restrictions other than assumptions A1-A4. We have looked at diseconomies of scope, constant marginal costs and independent firms. If we impose no such restrictions we might expect, from lemma 1, that prices could move in any direction as a reaction to a change in the exchange rate. As proposition 6 and proposition 7 will show, this is true "if and only if" firms are quite different. To see this, we first introduce a symmetry assumption, i.e. we will assume that firms' cost functions are identical, and that the exchange rate is 1. We call this the case of identical firms.

**Proposition 6.** *Given A1 - A4.2 the price in the market whose currency appreciates decreases and the other price increases if both firms are identical.*

PROOF:

With  $c_1 = c_2$  and  $e = 1$  the Cournot-Nash Equilibrium is symmetric with respect to firms. Thus we must have  $x_{1j}^* = x_{2j}^*$ ,  $j = 1, 2$ . This implies  $-\gamma_1(x^*) = \gamma_2(x^*) = \gamma(x^*)$ ,  $b_{ij}(x^*) = b_j(x^*)$   $i = 1, 2$  and  $c_i^r(x_{i1}^*, x_{i2}^*) = c^r(x_1^*, x_2^*)$   $i = 1, 2, r, s = 1, 2$ .

In the linear system (14) we now add up the first and the third equation and the second and the fourth equation. This reduces (14) to

$$\underbrace{\begin{bmatrix} a_{13} + a_{11} & , & a_{12} \\ a_{21} & , & a_{31} + a_{22} \end{bmatrix}}_A \begin{pmatrix} dX_1 \\ dX_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \gamma de.$$

Thus  $|\tilde{A}| \frac{dX_1}{\gamma de} = a_{31} + a_{22} + a_{12} < 0$  and  $|\tilde{A}| \frac{dX_2}{\gamma de} = -(a_{31} + a_{11} + a_{21}) > 0$ . Lastly, we must consider  $|\tilde{A}|$ .  $\tilde{A}$  inherits property A4.2 from A, thus  $|\tilde{A}| > 0$ , which completes the proof. ■

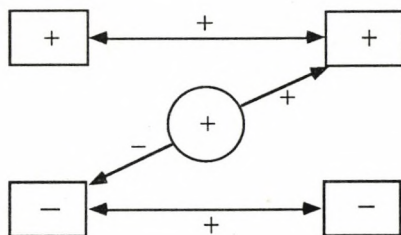


Figure 4

To show that, with general cost and demand functions, 'perverse' effects can occur, we therefore have to consider the case in which firms are different. To do this we once more concentrate on the direct and the cost effects only, ignoring strategic interaction. If cost functions have economies of scope we might guess from figure 4 that as a reaction to an increase in  $e$ , firm 1 would increase sales in both markets whereas firm 2 would decrease sales in both markets. If the economies of scope are strong enough to outweigh the direct effects, then  $P_2$  will increase and  $P_1$  will decrease!

Proposition 7 shows that this situation is compatible with A1 - A4.1. But the market chosen violates A4.2, which by itself does not have any justification.

**Proposition 7.** *With general cost and demand functions a market exists which satisfies A1 - A4.1 such that the price in the country whose currency appreciates increases and such that the price in the other country decreases.*

PROOF:

By Lemma 1 any parameters A,  $\gamma$  can be generated by a market satisfying A1 - A3. Now choose  $|b_{ij}|$   $i, j = 1, 2$  small. Since the solution of (14) is a continuous function in  $b_{ij}$  if  $|A_{(x^*)}| \neq 0$ , instead of taking the limit  $|b_{ij}| \rightarrow 0$  of the solution we can simply solve (14) for the limit case  $b_{ij} = 0$ .



The solution is

$$|A| \frac{dx_{11}}{de} = -\gamma_1 a_{12} |A_{22}| > 0, \quad |A| \frac{dx_{21}}{de} = \gamma_2 a_{44} |A_{11}| < 0,$$

$$|A| \frac{dx_{12}}{de} = \gamma_1 a_{11} |A_{22}| > 0, \quad |A| \frac{dx_{22}}{de} = -\gamma_2 a_{43} |A_{11}| < 0$$

Now choose  $-\gamma_1 |A_{22}| = \gamma_2 |A_{11}| = h > 0$  then  $|A| \frac{dX_1}{de} = h(a_{12} + a_{44})$  and  $|A| \frac{dX_2}{de} = -h(a_{43} + a_{11})$ . Next choose the economies of scope  $a_{12}, a_{43} > 0$  sufficiently large to outweigh the direct effects  $a_{44}, a_{11} < 0$ .

We remark that this choice is not compatible with A4.2, since then  $|a_{44}| < |a_{12}| < |a_{11}| < |a_{43}| < |a_{44}|$  a contradiction.

However it is compatible with stability of A. It is even compatible with A possessing the negative dominant diagonal property, i.e.  $a_{ii} < 0 \quad i = 1, \dots, 4$  and there exist weights  $w_j > 0, \quad j = 1, \dots, 4$  such that  $w_i |a_{ii}| > \sum_{j \neq i} w_j |a_{ij}| \quad i = 1, \dots, 4$ . To show this, since A is symmetric it is enough to show that A is negative definite. But this can always be assured by choosing  $a_{22}$  and  $a_{33}$  small enough, which completes the proof. ■

## 7. An Example

As we have noted, the effects of an exchange rate change are difficult to calculate as a result of the presence of economies of scale or scope. This is true even with linear demands, which limit the complications involved in calculating the impact of such changes.

To examine the type of change that may occur when the exchange rate is modified, we calculate the equilibrium, quantities, prices and profits for a simple example with linear demands and additively separable quadratic costs.

Specify the demand functions in market 1 and 2 as

$$P_1 = n_1(1 - x_{11} - x_{21})$$

$$P_2 = n_2(1 - x_{12} - x_{22}).$$

The cost functions are given by

$$c_1 = \alpha_1(x_{11} + x_{12}) - \frac{\beta_1}{2}(x_{11} + x_{12})^2$$

$$c_2 = \alpha_2(x_{21} + x_{22}) - \frac{\beta_2}{2}(x_{21} + x_{22})^2$$

The parameters  $n_i$ ,  $\alpha_i$ ,  $\beta_i$  are allowed to vary, and we note that when  $\beta_i < 0$  there are decreasing returns to scale, and the opposite is the case for  $\beta_i > 0$ . Linearity of the demand functions and additive separability of the cost functions reduce the complexity of the comparative statics problem considerably (compare with lemma 1 for example). Given these restrictions, we could not find perverse movements of prices, and to get even a surprising movement of prices, we had to assume diseconomies of scale for one and economies of scale for the other firm.

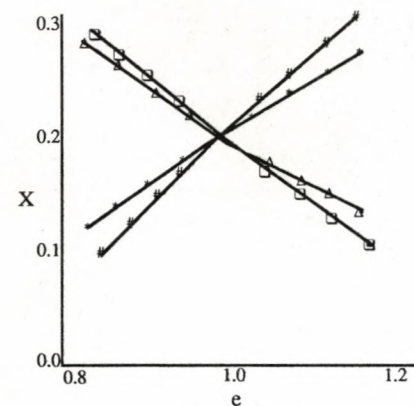
In the examples therefore, as we would expect from the theoretical part of our paper, we never have a case in which the indirect effects outweigh the direct effect.

Figure 5 shows a situation with identical firms both of which have economies of scale. In this case both indirect effects move in the same direction, and therefore  $x_{11}$  increases and  $x_{22}$  decreases. The exchange rate determines how the market is shared between the firms, and increasing  $e$  means increasing  $P_1$  and  $\Pi_1$  and decreasing  $P_2$  and  $\Pi_2$ .

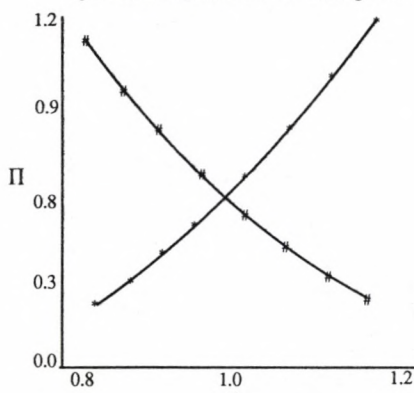
Figure 6 shows a situation in which both firms have diseconomies of scale. For exchange rates smaller than 1.2, the magnitude of the diseconomies determines the market shares. The substitution effects dominate the cost effects thus  $x_{11}$  increases and  $x_{22}$  decreases. Prices and profits show normal reactions.

The most interesting situation is given in figure 7. Whereas for firm 2 the cost effects always dominate the substitution effects, thus  $x_{22}$  decreases, for firm 1 initially the cost effects are stronger and the substitution effects dominate. Initially the decrease in  $x_{22}$  is compensated by an increase in  $x_{12}$ , for big enough exchange rates, firm 2's movement out of both markets determines the price change. It is also interesting to mention that figure 7 shows a situation with linear demand functions where both elasticities with respect to exchange rate changes become bigger than 1.

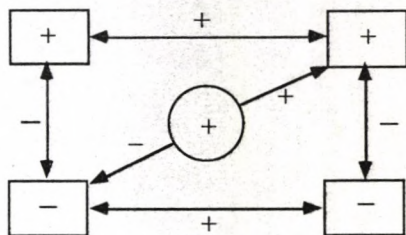
$$\alpha_1 = 5 = \alpha_2, \quad \beta_1 = 3 = \beta_2, \quad \eta_1 = 10 = \eta_2$$



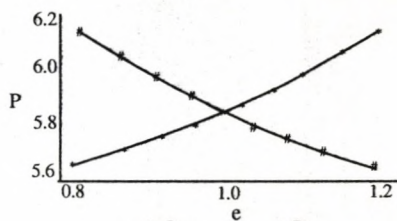
\*  $X_{11}$  #  $X_{12}$  □  $X_{21}$  △  $X_{22}$   
Quantities as a function of the exchange rate



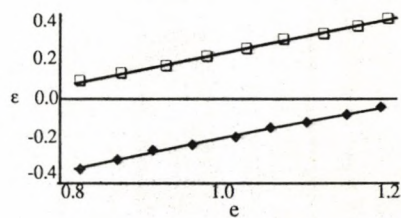
\*  $\Pi_1$  #  $\Pi_2$   
Profit as a function of the exchange rate



Direction of direct and indirect effects



\*  $P_1$  #  $P_2$   
Prices as a function of the exchange rate



□  $\epsilon_1$  ◆  $\epsilon_2$   
Elasticity of price with respect to exchange rate change

Figure 5



$$\alpha_1 = 5 = \alpha_2, \quad \beta_1 = -5, \quad \beta_2 = -1, \quad \eta_1 = 10 = \eta_2$$

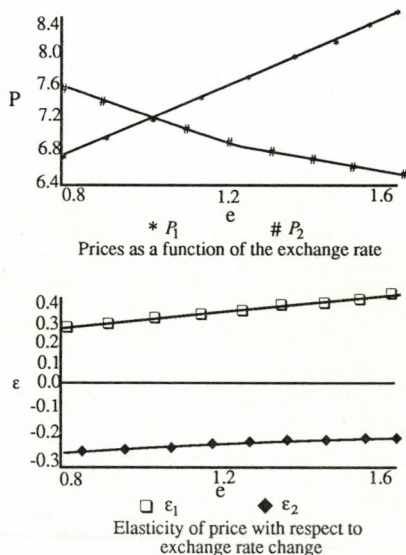
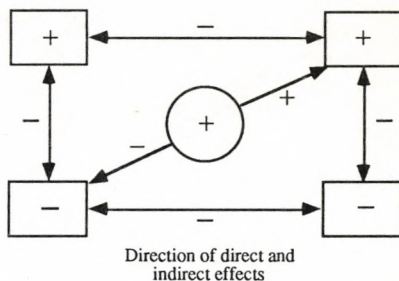
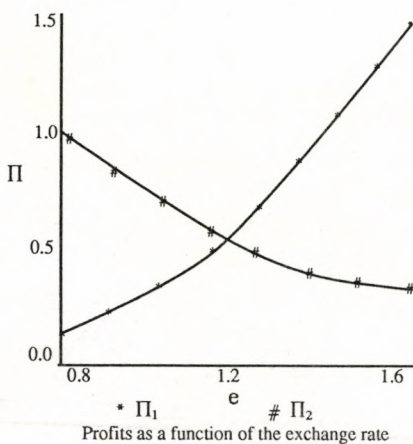
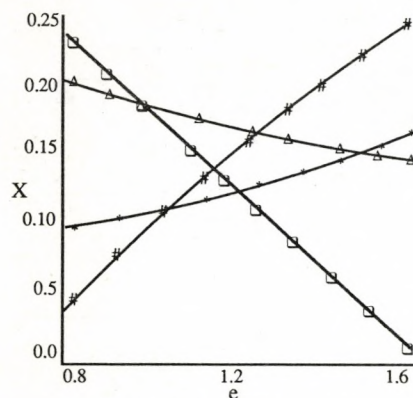
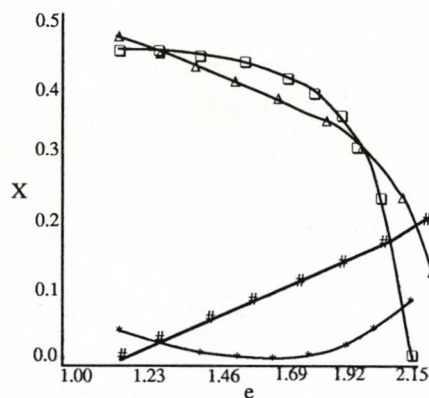
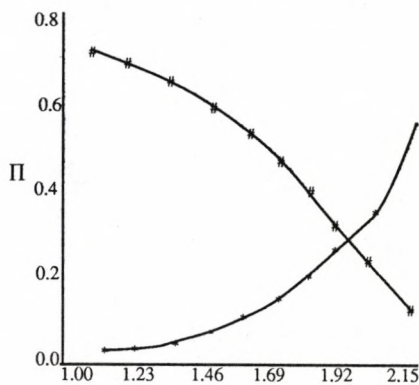


Figure 6

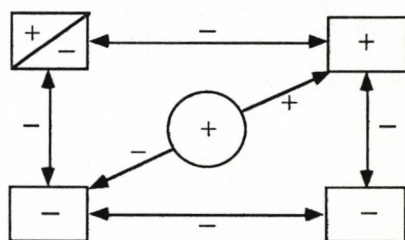
$$\alpha_1 = 3/2, \quad \alpha_2 = 2, \quad \beta_1 = -20/3, \quad \beta_2 = 2, \quad \eta_1 = 4, \quad \eta_2 = 10/3$$



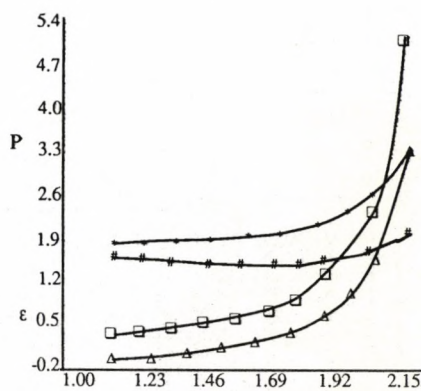
\*  $X_{11}$  #  $X_{12}$  □  $X_{21}$  △  $X_{22}$   
Quantities as a function of the exchange rate



\*  $\Pi_1$  #  $\Pi_2$   
Profits as a function of the exchange rate



Direction of direct and indirect effects



\*  $P_1$  #  $P_2$  e □  $\epsilon_1$  △  $\epsilon_2$   
Prices and elasticity of prices with respect to exchange rate change

Figure 7

## 8. Conclusions

In this paper we have examined a situation with two markets and duopolists, each based in one of two markets, but both selling in both markets. If the two markets are effectively separated and oligopolists can discriminate between markets, the effects of exchange rate changes on prices may be very complicated. The direction and magnitudes of these changes will depend critically on the extent of economies of scope and the strategic impacts of competitors' sales on their marginal profits.

We have given results showing that, if markets are effectively independent (constant marginal costs), or if firms are identical, then there is a simple monotonic relation between the prices in each market and the exchange rate. But with differing firms and general cost functions, both prices may change in the same direction and they may even increase in the market whose currency appreciates, and decrease in the market whose currency depreciates. Examples show that both prices may move in the same direction even if demand functions are linear and cost functions are additively separable.

This analysis helps to explain why the effects of exchange rate changes have been observed<sup>2</sup> to be incomplete and even perverse. We have not had to rely on dynamics,<sup>3</sup> inertia or sunk costs to obtain these results, they follow from simple comparative statics analysis. Thus, whilst the dynamics through which exchange rate changes are transmitted into prices are surely important, if market structure is taken into account they are not necessary to explain some commonly observed phenomena.

---

<sup>2</sup> See for example Kirman and Schueller (1990) and Mertens (1990).

<sup>3</sup> Froot and Klemperer (1989) show that dynamic effects may imply a price increase in the country whose currency appreciates.



## 9. References

- AUKRUST, O. (1977), "Inflation in the Open Economy: A Norwegian model," Artikler 96, Oslo: Statistik Sentralbyrå.
- BULOW, J.I., GEANAKOPOLOS, J.D. and P.D. KLEMPERER (1985), "Multi-market Oligopoly: Strategic Substitutes and Components," *Journal of Political Economy*, 93, pp. 488-511.
- DIXIT, A. (1986), "Comparative Statics for Oligopoly," *International Economic Review*, 27, pp. 107-122.
- DORNBUSCH, R. (1987), "Exchange Rates and Prices," *American Economic Review*, 77, pp. 93-106.
- FEENSTRA, R.C. (1989), "Symmetric Pass-Through of Tariffs and Exchange Rates under Imperfect Competition: An Empirical Test," *Journal of International Economics*, 27, pp. 24-45.
- FEINBERG, R.M. (1986), "The Interaction of Foreign Exchange and Market Power Effects on German Domestic Prices," *Journal of Industrial Economics*, XXXV, pp. 61-79.
- FISHER, E. (1989), "A Model of Exchange Rate Pass-Through," *Journal of International Economics*, 26, pp. 119-137.
- FROOT, K.A. and P.D. KLEMPERER (1989), "Exchange Rate Pass-Through when Market Share Matters," *American Economic Review*, 79, pp. 637-654.
- FRIEDMAN, J. (1977), *Oligopoly and the Theory of Games*, North-Holland, Amsterdam.
- GILBERT, C.L. (1991), "The Response of Primary Commodity Prices to Exchange Rate Changes," in L. Philips, ed., *Commodity, Futures and Financial Markets*, Kluwer Academic Publishers, Dordrecht, pp. 87-124.
- GIOVANNINI, A. (1988), "Exchange Rates and Traded Goods Prices," *Journal of International Economics*, 24, pp. 45-68.

JOHNSON, H.G. (1972), "The Monetary Approach to Balance of Payments Theory," in *Further Essays in Monetary Economics*, pp.229-249, George Allen and Unwin, London.

KIRMAN, A.P. and N. SCHUELLER (1990), "Price Leadership and Discrimination in the European Car Market," *Journal of Industrial Economics*, XXXIX, September, pp. 1-23.

KNETTER, M.M. (1989), "Exchange Rate Fluctuations and Price Discrimination by US and German Exporters," *American Economic Review*, 79, pp. 198-210.

KRUGMAN, P. (1987), "Pricing to Market when the Exchange Rate Changes," in S.W. ARNDT and J.D. RICHARDSON (eds.), *Real Financial Linkages among Open Economies*, M.I.T. Press, Cambridge, Mass.

MANN, C. (1986), "Prices, profit margins and exchange rates," *Federal Reserve Bulletin*, 72, pp. 366 - 379.

MERTENS, Y. (1990), "Modelling Price Behaviour in the European Car Market: 1970 - 1985," The Economics of Industry Group, Working Paper E I/1, STICERD, London School of Economics, London.

ROBINSON, W., WEBB, T.R. and M.A. TOWNSEND (1979), "The Influence of Exchange Rate Changes on Prices: A Study of Industrial Countries," *Economica*, 46, pp. 27-50.



# EUI WORKING PAPERS

EUI Working Papers are published and distributed by the  
European University Institute, Florence

Copies can be obtained free of charge – depending on the availability of  
stocks – from:

The Publications Officer  
European University Institute  
Badia Fiesolana  
I-50016 San Domenico di Fiesole (FI)  
Italy

Please use order form overleaf



# Publications of the European University Institute

To           The Publications Officer  
European University Institute  
Badia Fiesolana  
I-50016 San Domenico di Fiesole (FI)  
Italy

From       Name .....

Address .....

.....

.....

.....

.....

- ☐ Please send me a complete list of EUI Working Papers
- ☐ Please send me a complete list of EUI book publications
- ☐ Please send me the EUI brochure Academic Year 1990/91

Please send me the following EUI Working Paper(s):

No, Author .....

Title: .....

No, Author .....

Title: .....

No, Author .....

Title: .....

No, Author .....

Title: .....

Date .....

Signature .....



# **Working Papers of the Department of Economics** **Published since 1989**

**89/370**

B. BENSaid/R.J. GARY-BOBO/  
 S. FEDERBUSCH  
 The Strategic Aspects of Profit Sharing in the  
 Industry

**89/374**

Francisco S. TORRES  
 Small Countries and Exogenous Policy Shocks

**89/375**

Renzo DAVIDDI  
 Rouble Convertibility: A Realistic Target

**89/377**

Elettra AGLIARDI  
 On the Robustness of Contestability Theory

**89/378**

Stephen MARTIN  
 The Welfare Consequences of Transaction Costs  
 in Financial Markets

**89/381**

Susan SENIOR NELLO  
 Recent Developments in Relations Between the  
 EC and Eastern Europe

**89/382**

Jean GABSZEWICZ/ Paolo GARELLA/  
 Charles NOLLET  
 Spatial Price Competition With Uninformed  
 Buyers

**89/383**

Benedetto GUI  
 Beneficiary and Dominant Roles in  
 Organizations: The Case of Nonprofits

**89/384**

Agustín MARAVALL/ Daniel PEÑA  
 Missing Observations, Additive Outliers and  
 Inverse Autocorrelation Function

**89/385**

Stephen MARTIN  
 Product Differentiation and Market Performance  
 in Oligopoly

**89/386**

Dalia MARIN  
 Is the Export-Led Growth Hypothesis Valid for  
 Industrialized Countries?

**89/387**

Stephen MARTIN  
 Modeling Oligopolistic Interaction

**89/388**

Jean-Claude CHOURAQUI  
 The Conduct of Monetary Policy: What have we  
 Learned From Recent Experience

**89/390**

Corrado BENASSI  
 Imperfect Information and Financial Markets: A  
 General Equilibrium Model

**89/394**

Serge-Christophe KOLM  
 Adequacy, Equity and Fundamental Dominance:  
 Unanimous and Comparable Allocations in  
 Rational Social Choice, with Applications to  
 Marriage and Wages

**89/395**

Daniel HEYMANN/ Axel LEIJONHUFVUD  
 On the Use of Currency Reform in Inflation  
 Stabilization

**89/400**

Robert J. GARY-BOBO  
 On the Existence of Equilibrium Configurations  
 in a Class of Asymmetric Market Entry Games \*

**89/402**

Stephen MARTIN  
 Direct Foreign Investment in The United States

**89/413**

Francisco S. TORRES  
 Portugal, the EMS and 1992: Stabilization and  
 Liberalization

**89/416**

Joerg MAYER  
 Reserve Switches and Exchange-Rate Variability:  
 The Presumed Inherent Instability of the  
 Multiple Reserve-Currency System

**89/417**

José P. ESPERANÇA/ Neil KAY  
 Foreign Direct Investment and Competition in  
 the Advertising Sector: The Italian Case

\* Working Paper out of print

**89/418**

Luigi BRIGHI/ Mario FORNI  
Aggregation Across Agents in Demand Systems

**89/420**

Corrado BENASSI  
A Competitive Model of Credit Intermediation

**89/422**

Marcus MILLER/ Mark SALMON  
When does Coordination pay?

**89/423**

Marcus MILLER/ Mark SALMON/  
Alan SUTHERLAND  
Time Consistency, Discounting and the Returns  
to Cooperation

**89/424**

Frank CRITCHLEY/ Paul MARRIOTT/  
Mark SALMON  
On the Differential Geometry of the Wald Test  
with Nonlinear Restrictions

**89/425**

Peter J. HAMMOND  
On the Impossibility of Perfect Capital Markets

**89/426**

Peter J. HAMMOND  
Perfect Option Markets in Economies with  
Adverse Selection

**89/427**

Peter J. HAMMOND  
Irreducibility, Resource Relatedness, and Survival  
with Individual Non-Convexities

\* \* \*

**ECO No. 90/1\*\***

Tamer BAŞAR and Mark SALMON  
Credibility and the Value of Information  
Transmission in a Model of Monetary Policy  
and Inflation

**ECO No. 90/2**

Horst UNGERER  
The EMS – The First Ten Years  
Policies – Developments – Evolution

**ECO No. 90/3**

Peter J. HAMMOND  
Interpersonal Comparisons of Utility: Why and  
how they are and should be made

**ECO No. 90/4**

Peter J. HAMMOND  
A Revelation Principle for (Boundedly) Bayesian  
Rationalizable Strategies

**ECO No. 90/5**

Peter J. HAMMOND  
Independence of Irrelevant Interpersonal  
Comparisons

**ECO No. 90/6**

Hal R. VARIAN  
A Solution to the Problem of Externalities and  
Public Goods when Agents are Well-Informed

**ECO No. 90/7**

Hal R. VARIAN  
Sequential Provision of Public Goods

**ECO No. 90/8**

T. BRIANZA, L. PHILIPS and J.F. RICHARD  
Futures Markets, Speculation and Monopoly  
Pricing

**ECO No. 90/9**

Anthony B. ATKINSON/ John  
MICKLEWRIGHT  
Unemployment Compensation and Labour  
Market Transition: A Critical Review

**ECO No. 90/10**

Peter J. HAMMOND  
The Role of Information in Economics

**ECO No. 90/11**

Nicos M. CHRISTODOULAKIS  
Debt Dynamics in a Small Open Economy

**ECO No. 90/12**

Stephen C. SMITH  
On the Economic Rationale for Codetermination  
Law

**ECO No. 90/13**

Elettra AGLIARDI  
Learning by Doing and Market Structures

**ECO No. 90/14**

Peter J. HAMMOND  
Intertemporal Objectives

**ECO No. 90/15**

Andrew EVANS/Stephen MARTIN  
Socially Acceptable Distortion of Competition:  
EC Policy on State Aid

\*\* Please note: As from January 1990, the EUI Working Papers Series is divided into six sub-series, each series will be numbered individually (e.g. EUI Working Paper LAW No. 90/1).



**ECO No. 90/16**  
Stephen MARTIN  
Fringe Size and Cartel Stability

**ECO No. 90/17**  
John MICKLEWRIGHT  
Why Do Less Than a Quarter of the  
Unemployed in Britain Receive Unemployment  
Insurance?

**ECO No. 90/18**  
Mrudula A. PATEL  
Optimal Life Cycle Saving  
With Borrowing Constraints:  
A Graphical Solution

**ECO No. 90/19**  
Peter J. HAMMOND  
Money Metric Measures of Individual and Social  
Welfare Allowing for Environmental  
Externalities

**ECO No. 90/20**  
Louis PHILIPS/  
Ronald M. HARSTAD  
Oligopolistic Manipulation of Spot Markets and  
the Timing of Futures Market Speculation

**ECO No. 90/21**  
Christian DUSTMANN  
Earnings Adjustment of Temporary Migrants

**ECO No. 90/22**  
John MICKLEWRIGHT  
The Reform of Unemployment Compensation:  
Choices for East and West

**ECO No. 90/23**  
Joerg MAYER  
U. S. Dollar and Deutschmark as Reserve Assets

**ECO No. 90/24**  
Sheila MARNIE  
Labour Market Reform in the USSR:  
Fact or Fiction?

**ECO No. 90/25**  
Peter JENSEN/  
Niels WESTERGÅRD-NIELSEN  
Temporary Layoffs and the Duration of  
Unemployment: An Empirical Analysis

**ECO No. 90/26**  
Stephan L. KALB  
Market-Led Approaches to European Monetary  
Union in the Light of a Legal Restrictions  
Theory of Money

**ECO No. 90/27**  
Robert J. WALDMANN  
Implausible Results or Implausible Data?  
Anomalies in the Construction of Value Added  
Data and Implications for Estimates of Price-  
Cost Markups

**ECO No. 90/28**  
Stephen MARTIN  
Periodic Model Changes in Oligopoly

**ECO No. 90/29**  
Nicos CHRISTODOULAKIS/  
Martin WEALE  
Imperfect Competition in an Open Economy

\* \* \*

**ECO No. 91/30**  
Steve ALPERN/Dennis J. SNOWER  
Unemployment Through 'Learning From  
Experience'

**ECO No. 91/31**  
David M. PRESCOTT/Thanasis STENGOS  
Testing for Forecastable Nonlinear Dependence  
in Weekly Gold Rates of Return

**ECO No. 91/32**  
Peter J. HAMMOND  
Harsanyi's Utilitarian Theorem:  
A Simpler Proof and Some Ethical  
Connotations

**ECO No. 91/33**  
Anthony B. ATKINSON/  
John MICKLEWRIGHT  
Economic Transformation in Eastern Europe  
and the Distribution of Income

**ECO No. 91/34**  
Svend ALBAEK  
On Nash and Stackelberg Equilibria when Costs  
are Private Information

**ECO No. 91/35**  
Stephen MARTIN  
Private and Social Incentives  
to Form R & D Joint Ventures

**ECO No. 91/36**  
Louis PHILIPS  
Manipulation of Crude Oil Futures

**ECO No. 91/37**  
Xavier CALSAMIGLIA/Alan KIRMAN  
A Unique Informationally Efficient and  
Decentralized Mechanism With Fair Outcomes

**ECO No. 91/38**

George S. ALOGOSKOUFIS/  
Thanasis STENGOS  
Testing for Nonlinear Dynamics in Historical  
Unemployment Series

**ECO No. 91/39**

Peter J. HAMMOND  
The Moral Status of Profits and Other Rewards:  
A Perspective From Modern Welfare Economics

**ECO No. 91/40**

Vincent BROUSSEAU/Alan KIRMAN  
The Dynamics of Learning  
in Mis-Specified Models

**ECO No. 91/41**

Robert James WALDMANN  
Assessing the Relative Sizes of Industry- and  
Nation Specific Shocks to Output

**ECO No. 91/42**

Thorsten HENS/Alan KIRMAN/Louis PHILIPS  
Exchange Rates and Oligopoly

**ECO No. 91/43**

Peter J. HAMMOND  
Consequentialist Decision Theory and  
Utilitarian Ethics







© The Author(s). European University Institute.

Digitised version produced by the EUI Library in 2020. Available Open Access on Cadmus, European University Institute Research Repository.





